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HEAT TRANSFER IN VERTICAL UPWARDS GAS–LIQUID SLUG FLOW

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Abstract—An analytical solution for the transient behaviour of heat transfer in vertical upward gas–liquid slug flow is presented. The results allow the prediction of the temperature variation with time and location and the wall temperature fluctuations, as well as the average heat transfer coefficients.

NOMENCLATURE

- a constant defined in equations (8) and (9)
- A cross-sectional area of the pipe
- A_G cross-sectional area of the Taylor bubble
- A_f cross-sectional area of the liquid film
- C_L constant in the friction function correlation
- C_p specific heat
- f friction factor
- g acceleration of gravity
- h heat transfer coefficient
- k thermal conductivity
- L_s length of the liquid slug region
- L_B length of the bubble, or the film region
- m exponent, equation (4)
- n number of slug units
- Q_G volumetric flow rate of gas
- Q_t total volumetric flow rate
- q constant heat flux
- T liquid temperature
- T_G gas temperature
- T_w wall temperature
- ΔT_p defined in equation (26)
- t time
- t_{mf} time needed for a liquid particle to move from the bubble front to its tail
- t_{ms} time needed for a liquid particle to move from the slug front to its tail
- t_{mp} time needed for a liquid particle to move from one bubble front to the next bubble front
- t_{sf} time period during which a film is seen at fixed location
- t_{ss} time period during which a liquid slug is seen at fixed location
- t_{sp} time period during which a slug unit is seen at a fixed location
- t_{in} time when the liquid particle enters the heated section the first time
- t_{in2} time of the second entrance to the heated section
- V_f absolute film velocity
- V_G gas bubble velocity, translational velocity
- V_s slug velocity
- X distance from the entrance to the heated section
- x axial location relative to the moving gas bubble
- X_p the distance that a liquid particle moves during t_{mp} [equation (22)]

Greek symbols
- α constant defined in equation (52)
- β constant defined in equation (53)
- ρ density
- ν kinematic viscosity
- θ_{ms} defined by equation (25)
- θ_{mf} defined by equation (30)

Subscripts and superscripts
- B bubble
- f film
- G gas
- L liquid
- p period
- s slug
- $\bar{}$ average

1. INTRODUCTION

HEAT transfer during two-phase gas–liquid flow takes place in many industrial processes. It occurs in condensers, boilers and refrigerators or in the simultaneous heating of two phases such as the preheating of liquid and gas feed to a reactor or in natural gas and condensate in pipe lines.

The mechanism of heat transfer can change markedly according to the flow regimes during the gas–liquid flow, but most of the reports on heat transfer data do not consider flow regimes at all and use very general correlations for the average heat transfer coefficients in two-phase flow [1].

Prediction of time varying temperature and heat flux in vertical upward gas–liquid slug flow which takes into account the unsteady nature of slug flow is as yet an unresolved problem. Although extensive research on heat transfer in two-phase flow has been carried out, experimental data as well as physical models are still limited, especially for slug flow.

Some investigators dealt with heat transfer during horizontal slug flow [2–4] but their results were presented as time average data although the intermittent flow is an unsteady state process.

Niu [5] developed a numerical model for heat transfer during gas–liquid slug flow in horizontal tubes. The model is capable of calculating the time and position dependence of temperatures of the fluids and the wall as well as the heat flux provided the local heat transfer coefficients are given.

Shaharabanny [6] measured the local heat transfer coefficient in horizontal slug flow and compared the temperature and heat flux fluctuations with Niu [5]. No experimental data and theory are available on transient heat transfer in upward gas–liquid slug flow.

It is the objective of this work to suggest a model and an analytical solution for the transient behaviour of heat transfer in vertical upward two-phase slug flow.

2. ANALYSIS

The purpose of this analysis is to solve the transient heat transfer characteristics in upward slug flow. Since slug flow is intermittent in the axial direction, heat flux and temperature will vary with time in the fluids as well as in the wall. The heat flux from the wall into the liquid has a different character when a liquid slug is in contact with the wall or when the liquid film adjacent to the Taylor bubble is in contact with the wall. In addition, the convection changes direction from upward flow in the liquid slug to downward flow in the film. As a result, the process of heat transfer to upward slug flow has an interesting periodic character influencing the local temperature and heat flux fluctuations as well as the average heat transfer in slug flow.

Heat transfer into slug flow requires the knowledge of the hydrodynamic character of slug flow. Parameters such as the bubble, slug and film velocities, as well as the film thickness, should be known. Thus the first step in this solution is to determine the hydrodynamics parameters via a simple hydrodynamic model that permits the determination of the slug flow parameters. In the second step the analysis of the heat transfer is carried out using the hydrodynamic parameters, as the heat transfer coefficients are based on the hydrodynamic conditions. Based on the detailed variation of the temperature profile with time and space wall temperature, fluctuations as well as average heat transfer coefficients are calculated.

Hydrodynamics

The system of two-phase slug flow is shown in Fig. 1. It is assumed that the gas is located in large 'Taylor bubbles' which have a diameter almost equal to the pipe diameter, a length L_B and which move uniformly upward with a velocity V_G. The bubbles are assumed to have cylindrical shape with constant cross-sectional area. The Taylor bubbles are separated by liquid slugs of length L_s which bridge the pipe. The liquid within the slug moves upward with a velocity V_s. Between the Taylor bubble and the pipe wall the liquid flows downward in a thin film with an average velocity V_f and a cross-sectional area A_f. For the solution of the heat transfer problem in slug flow the hydrodynamic parameters V_G, V_s, V_f, L_B, L_s and A_f are needed. V_G is also the translational velocity of the slug system as seen to an outside observer.

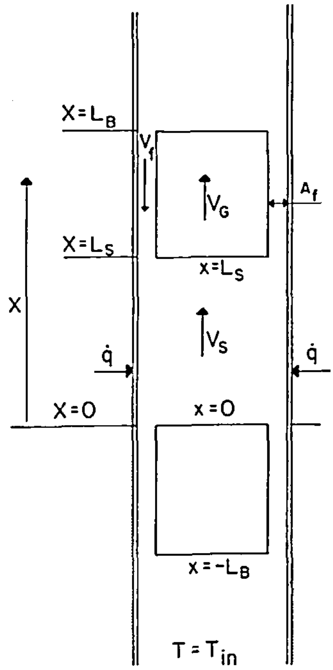


FIG. 1. The slug flow model.

The velocity of the Taylor bubble is given by the relation [7]

$$V_G = 1.2\,V_s + 0.35(gD)^{1/2}. \tag{1}$$

The total input volumetric rate is constant at any cross-section, therefore, is given by

$$Q_t = V_s A. \tag{2}$$

A liquid mass-balance relative to the moving gas bubble yields

$$A(V_G - V_s) = A_f(V_G + V_f). \tag{3}$$

The liquid film adjacent to the gas bubble is assumed to behave as a free-falling film. The film average velocity V_f is related to its cross-sectional area, A_f, by the force balance between gravity and wall shear,

$$f(\tfrac{1}{2}\rho_L V_f^2)\pi D = \rho_L g A_f \tag{4}$$

where

$$f = C_L\left(\frac{4A_f V_f}{\pi D \nu_L}\right)^{-m}$$

for turbulent flow, $C_L = 0.046$ and $m = 0.2$. Equations (1)–(4) consist of a set of four independent equations for the unknowns A_f, V_s, V_f and V_G.

Experimental observations for the air–water system suggest that the stable liquid slug has a length, L_s, which is almost constant and independent of the gas and liquid flow rates [8, 9] and is normally of the order of 16*D*.

The ratio $L_B/(L_B+L_s)$ can be derived from a simple mass balance that yields

$$L_B/(L_B+L_s) = Q_G/(V_G A_G). \tag{5}$$

Heat transfer

It is assumed that the pipe is heated in a region where fully developed slug flow exists. The inlet temperature to the heated section is T_{in}. Thermal energy balances for the film zone yields

$$\frac{\partial T}{\partial t} = V_f \frac{\partial T}{\partial X} + \frac{\pi D h_f}{A_f \rho_L C_{p_L}}(T_w - T) \tag{6}$$

and for the liquid slug zone

$$\frac{\partial T}{\partial t} = -V_s \frac{\partial T}{\partial X} + \frac{\pi D h_s}{A \rho_L C_{p_L}}(T_w - T) \tag{7}$$

where X designates the distance from the entrance to the heated section (Fig. 1) and T is the average liquid temperature in any cross-section. The transfer of thermal energy through the interface between gas and liquid is neglected here.

The problem now is to predict the axial temperature profile and the time–temperature variation for the liquid in the slug and the film and the wall temperature or heat flux, under given heating conditions. In the analysis two cases are considered. The first case is of constant heat flux while in the second one, the wall temperature is assumed constant. For the case of constant heat flux, equations (6) and (7) take the form

$$\frac{\partial T}{\partial t} = V_f \frac{\partial T}{\partial X} + \frac{aq}{A_f} \tag{8}$$

for the liquid film, and

$$\frac{\partial T}{\partial t} = -V_s \frac{\partial T}{\partial X} + \frac{aq}{A} \tag{9}$$

for the liquid slug, where

$$a = \pi D/\rho_L C_{p_L}.$$

At $t = 0$ we locate the front of a Taylor bubble at the entrance to the heating section ($X = 0$) (Fig. 1).

Integrating the differential equations (8) and (9) by the method of characteristics yields for the film zone

$$V_f t + X = C_1 \quad \text{(the characteristic line)}, \tag{10a}$$

$$\frac{aq}{A_f} t - T = C_2 = f(V_f t + X), \tag{10b}$$

and for the liquid slug zone

$$-V_s t + X = C_1', \tag{11a}$$

$$\frac{aq}{A} t - T = C_2' = f(-V_s t + X). \tag{11b}$$

The above relations state that the function $(aqt/A_f - T)$ is constant for a moving liquid element ($V = V_f$) in the film zone and similarly the function $(aqt/A - T)$ is constant for the slug zone for a point moving with the fluid at a velocity V_s.

The axial location relative to the moving gas bubble is designated by x. The lines x = const in Fig. 2 describe the translational motion of the front and tail of the Taylor bubbles that move along the pipe while X designates the axial distance from the entrance to the heated section. At $t = 0$, $x = X = 0$. The line $x = 0$

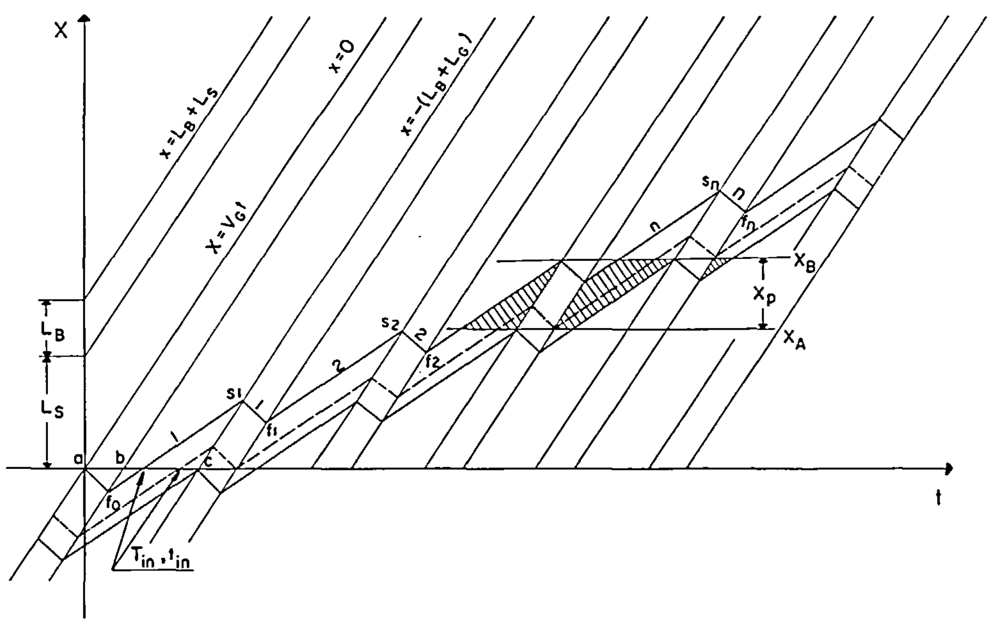


FIG. 2. Solution by the method of characteristics.

($X = V_G t$) yields the location of the front of the Taylor bubble that at $t = 0$ is located at $X = 0$, the line $x = -L_B$ ($X = -L_B + V_G t$) represents the location of the bottom of this bubble and so on for the next slug units. The region between front and tail of a Taylor bubble is a region in which a liquid film exists while the region between the tail and the next front corresponds to a liquid slug region.

A fluid particle moves downwards when it is within the film zone and changes its direction of motion when it enters the liquid slug that moves upward. The trajectories (10b) and (11b) of a fluid particle flowing through the pipe are also described in Fig. 2. The upper solid line describes the trajectory of a particle that at $t = 0$ is located exactly at the entrance ($X = 0$) and is in front of a Taylor bubble. The lower solid line is the trajectory of a particle that is at the bubble front of the next bubble when it reaches the point $X = 0$. The area between these two lines includes the trajectories of all the particles that reach the entrance during one slug period, namely the time that a slug unit is seen to pass a fixed location.

Since the liquid moves forward only in the liquid slug any liquid particle enters the heated section only when it is within the liquid slug. After entering the heated section a liquid particle may stay in it but it can also leave the heated section while moving downwards with the liquid film and enter the heated section with the next upcoming slug. Note also that this process may occur several times depending on the location of the liquid particle in the slug, on its first entrance into the heated section and on the hydrodynamic parameters of the slug flow. In Fig. 2 all the trajectories between the dashed line and lower line represents the passage of such particles that enter the heated section twice. Equations (10) and (11) are applicable only in the range where a film and a slug exist respectively. Consider, for example the upper characteristic line in Fig. 2. At $t = 0$ the front of the bubble, $x = 0$, is located at $X = 0$. Equation (10a) is the characteristic line $a - f_0$. At point f_0 the liquid particle enters the slug zone and the characteristic curve is represented by the line $f_0 - s_1$, [equation (11a)], and so on. The location along the characteristic curve in the bottom of the first slug that enters the heated section is designated s_1 (for all the characteristic trajectories of particles that enter the pipe in one slug period). The bottom of the film that follows is designated as f_1, and so on (Fig. 2).

The characteristic curve represents the actual physical trajectory of a liquid particle passing the heated pipe. This trajectory for a particle that at $t = 0$ is at $X = 0$ can be visualized, for example, as follows: The liquid particle flows with the film downwards at a velocity of V_f, at the same time the bubble moves upward. When the particle arrives at the bottom of the bubble it begins to move upward with the slug with a velocity of V_s until the next bubble overtakes the liquid particle at which point it moves downwards with the next film and so on.

The time period during which a film zone passes a fixed location is designated as t_{sf} where (Fig. 2)

$$t_{sf} = \frac{L_B}{V_G} = t_b - t_a. \tag{12}$$

Likewise for the slug

$$t_{ss} = \frac{L_s}{V_G} = t_c - t_b \tag{13}$$

while

$$t_{sp} = t_{sf} + t_{ss} = \frac{L_s + L_B}{V_G}. \tag{14}$$

The time needed for a particle to move from the bubble front to the bubble tail is

$$t_{mf} = \frac{L_B}{V_G + V_f} = t_{f1} - t_{s1} \tag{15}$$

likewise for the slug

$$t_{ms} = \frac{L_s}{V_G - V_s} = t_{s2} - t_{f1} \tag{16}$$

and

$$t_{mp} = t_{ms} + t_{mf} = t_{s2} - t_{s1}. \tag{17}$$

Figure 2 contains the trajectories of 3 typical particles. The region between the upper and the lower curves contains the trajectories of all the particles that enter the heated se...on in the first slug period ($0 < t < t_{sp}$). The region between the upper and the dashed curve contains all the trajectories of the particles that enter the heated section only once, whereas the region between the dashed and the lower solid curve contains the trajectories of the particles that enter the heated section twice. The time from $t = 0$ to the time that the liquid particle enters the heated section the first time is t_{in}. The criteria for the region above the dashed trajectory is

$$t_{mf}(1 + V_f/V_s) \leqslant t_{in} \leqslant t_{sp} + t_{sf} - t_{mf}(1 + V_f/V_s) \tag{18}$$

whereas below the dashed curve is

$$t_{sp} + t_{sf} - t_{mf}(1 + V_f/V_s) < t_{in} \leqslant t_{sp}. \tag{19}$$

A solution of the temperature profile is performed along the characteristic lines (the trajectories lines) for any given trajectory.

The solution for the trajectories bounded by the upper and dashed curves. For the slug zone

$$t_{f(n-1)} \leqslant t \leqslant t_{sn}, \quad n \geqslant 2,$$

$$T = \frac{aq}{A}\theta_{ms} + (n-2)\Delta T_p + \frac{aq}{A_f}t_{mf} + \frac{aq}{A}(t_{s1} - t_{in}) + T_{in}, \tag{20}$$

$$X = X_{s1} + (n-2)X_p - V_f t_{mf} + V_s \theta_{ms} \tag{21}$$

where n designates the slug number, $n = 1$ for the first slug that enters the heated section, $n = 2$ for the second slug, etc. Likewise, the film following the first slug is designated by $n = 1$ and so on. s_1 is the point at the bottom of the first slug, f_1 is the point at the bottom of the first film ($n = 1$) and so on (Fig. 2).

X_p is the distance that a particle moves during t_{mp} and is given by

$$X_p = t_{ms}V_s - t_{mf}V_f. \quad (22)$$

X_{s1} is the X location of s_1 which depends on t_{in} of the considered characteristic line

$$X_{s1} = \frac{V_G V_s}{V_G - V_s}(t_{sp} - t_{in}). \quad (23)$$

t_{sn} is the time at s_n and t_{fn} is the time at f_n,

$$t_{s1} = (V_G t_{sp} - V_s t_{in})/(V_G - V_s), \quad (24)$$

$$\theta_{ms} = t - t_{f(n-1)}. \quad (25)$$

ΔT_p, the increase of temperatures along one moving period is

$$\Delta T_p = \frac{aq}{A_f}t_{mf} + \frac{aq}{A}t_{ms}. \quad (26)$$

The solution for the first slug needs special treatment and is given by $t_{in} \leqslant t \leqslant t_{s1}$, $n = 1$,

$$T = \frac{aq}{A}(t - t_{in}) + T_{in}, \quad (27)$$

$$X = V_s(t - t_{in}). \quad (28)$$

For the liquid film zone

$$t_{sn} < t < t_{fn}, \quad n \geqslant 1,$$

$$T = \frac{aq}{A_f}\theta_{mf} + (n-1)\Delta T_p + \frac{aq}{A}(t_{s1} - t_{in}) + T_{in}, \quad (29)$$

$$X = X_{s1} + (n-1)X_p - V_f\theta_{mf} \quad (30)$$

where

$$\theta_{mf} = t - t_{sn}.$$

The solution for the trajectories bounded by the dashed and lower solid lines (particles that enter the heated section twice). For the liquid slug zone

$$t_{f(n-1)} \leqslant t \leqslant t_{sn}, \quad n \geqslant 3,$$

$$T = \frac{aq}{A}\theta_{ms} + (n-3)\Delta T_p + \frac{aq}{A_f}t_{mf} + \frac{aq}{A}(t_{s2} - t_{in2}) + T_{in2}, \quad (31)$$

$$X = X_{s2} + (n-3)X_p - V_f t_{mf} + V_s\theta_{ms} \quad (32)$$

where

$$t_{s2} = (2V_G t_{sp} - V_s t_{in2})/(V_G - V_s),$$

$$X_{s2} = V_s(2V_G t_{sp} - V_G t_{in2})/(V_G - V_s), \quad (33)$$

$$t_{in2} = t_{in} + t_{mf}(1 + V_f/V_s). \quad (34)$$

T_{in2} is the inlet temperature on its second entrance into the heated pipe. It is assumed here that the pipe below the heated section is insulated and thus the temperature of the particle in the film when it leaves the heated section is

$$T_{in2} = \frac{aqV_G}{(V_G - V_s)V_f A_f A}(t_{sp} - t_{in})(V_s A + V_f A_f) + T_{in}. \quad (35)$$

For the first and second slug the solution is

$$T = \frac{aq}{A}(t - t_{in}) + T_{in}, \quad t_{in} \leqslant t \leqslant t_{s1}, \quad n = 1, \quad (36)$$

$$X = V_s(t - t_{in}), \quad (37)$$

$$T = \frac{aq}{A}(t - t_{in2}) + T_{in2}, \quad t_{in2} \leqslant t \leqslant t_{s2}, \quad n = 2, \quad (38)$$

$$X = V_s(t - t_{in2}). \quad (39)$$

For the film zone

$$t_{sn} < t < t_{fn}, \quad n \geqslant 2,$$

$$T = \frac{aq}{A_f}\theta_{mf} + (n-2)\Delta T_p + \frac{aq}{A}(t_{s2} - t_{in2}) + T_{in2}, \quad (40)$$

$$X = X_{s2} + (n-2)X_p - V_f\theta_{mf}. \quad (41)$$

For the first film the solution is

$$T = \frac{aq}{A}(t_{s1} - t_{in}) + \frac{aq}{A_f}\theta_{mf} + T_{in}, \quad (42)$$

$$X = X_{s1} - V_f(t - t_{s1}), \quad x \geqslant 0. \quad (43)$$

The above equations relate T and X to t along a characteristic line which also depends on t_{in}, namely

$$X = X(t, t_{in}),$$

$$T = T(t, t_{in}).$$

n is also a parameter in these equations that can be expressed as a function of t and t_{in},

$$n = \text{integer}\,[(t - t_{f0})/t_{mp}] + 1, \quad t_{f0} = \frac{t_{sf}V_G - t_{in}V_s}{V_G - V_s} \quad (44)$$

where t_{f0} is the intersection of the characteristic curves with the line $x = -L_B$ (Fig. 2).

Note that, since the temperature variation with time is periodic,

$$T(X, t) = T(X, t + t_{sp}). \quad (45)$$

Knowing the temperature behaviour along the characteristic curves, the problem now is to determine $T(X_1, t_1)$ at any given point (X_1, t_1).

Thus, for a given location X_1 that has a value say between X_A and X_B (Fig. 2) one needs to find t^* which is within the cross hatched area and is t_{sp} away from t_1. Note that because the slope of this bounded region changes direction, the value of t^*, as well as n, is not uniquely confined to a single slug unit but can be placed in 3 different slug units depending on the exact position of X between X_A and X_B. Likewise, is the case for the film zone. The procedure for finding $T(X, t)$ of any arbitrary point (X, t) includes the following steps:

(a) It is checked whether the solution lies in the slug or film zone and θ_{ms} or θ_{mf} are found.

(b) n, t^* and t_{in} are found.

For the case of constant wall temperature, the characteristic curves are exactly the same as for the case of constant heat flux. Solving equations (6) and (7) for the case of constant wall temperature yields: for the film zone,

$$(T_w - T)\exp(ah_f t/A_f) = C_2 \qquad (46)$$

along

$$V_f t + X = C_1 \qquad (47)$$

and for the slug zone

$$(T_w - T)\exp(ah_s t/A) = C_2' \qquad (48)$$

along

$$X = V_s t = C_1'. \qquad (49)$$

The solution along any trajectory bounded by the solid upper and dashed curves (Fig. 2). For the slug zone

$$t_{f(n-1)} \leqslant t \leqslant t_{sn}, \quad n \geqslant 2,$$

$$T = T_w - (T_w - T_{in}) \mathrm{e}^{-[\alpha(t_{s1} - t_{in}) + \alpha\theta_{ms} + (n-2)\alpha t_{ms} + (n-1)\beta t_{mf}]}, \qquad (50)$$

and for the film zone

$$t_{sn} < t < t_{fn} \quad n \geqslant 1,$$

$$T = T_w - (T_w - T_{in}) \mathrm{e}^{-[\alpha(t_{s1} - t_{in}) + \beta\theta_{mf} + (n-1)\alpha t_{ms} + (n-1)\beta t_{mf}]} \qquad (51)$$

where

$$\alpha = ah_s/A, \qquad (52)$$

$$\beta = ah_f/A_f, \qquad (53)$$

h_s and h_f are the heat transfer coefficients of the slug and liquid film region, respectively.

The solution along any trajectory bounded by the dashed and lower solid lines. For the liquid slug zone

$$t_{f(n-1)} \leqslant t \leqslant t_{sn}, \quad n \geqslant 3,$$

$$T = T_w - (T_w - T_{in2}) \mathrm{e}^{-[\alpha(t_{s2} - t_{in2}) + \alpha\theta_{ms} + (n-2)\beta t_{mf} + (n-3)\alpha t_{ms}]}, \qquad (54)$$

and for the liquid film zone

$$t_{sn} < t < t_{fn}, \quad n \geqslant 2,$$

$$T = T_w - (T_w - T_{in2}) \mathrm{e}^{-[\alpha(t_{s2} - t_{in2}) + \beta\theta_{mf} + (n-2)\alpha t_{ms} + (n-2)\beta t_{mf}]} \qquad (55)$$

where

$$T_{in2} = T_w - (T_w - T_{in}) \mathrm{e}^{-\{\alpha(t_{s1} - t_{in}) + [\beta V_G V_s/(V_G - V_s)](t_{sp} - t_{in})/V_f\}} \qquad (56)$$

assuming the pipe below the heated section is insulated.

3. RESULTS AND DISCUSSION

Some typical results for temperature variation with position and time were calculated.

A case of air–water upward slug flow in a 0.0254 m dia. pipe with liquid and gas superficial velocities of 0.25 m s^{-1} was considered. The hydrodynamic parameters that were obtained by the solution of the hydrodynamic model are:

$$V_G = 0.778 \text{ m s}^{-1}, \quad V_s = 0.502 \text{ m s}^{-1},$$

$$V_f = 1.37 \text{ m s}^{-1}, \quad L_s = 0.4064 \text{ m},$$

$$L_B = 0.2387 \text{ m}, \quad A_f = 0.648 \times 10^{-4} \text{ m}^2.$$

The local heat transfer coefficients for air and water (both in the film and the slug) were calculated according to Colburn's [10] correlation,

$$\frac{hd}{k} = 0.023\left(\frac{\rho d V}{\mu}\right)^{0.8}\left(\frac{C_p \mu}{k}\right)^{0.33} \qquad (57)$$

where d is the hydraulic diameter.

Figure 3 shows the variation of the liquid temperature with time for given locations along one slug unit, for the case of constant heat flux. The temperature variation at any fixed location is periodic with a cycle time of t_{sp}. A more convenient representation of the local temperature as a function of time is given in Fig. 4 using the coordinates of the temperature vs $t - (X - X_0)/V_G$ (where X_0 is the location of the bubble front). This representation enables us to compare the temperature variation at the same position relative to the slug unit. The interesting result is that the temperature variation at each location is not repeated, despite the fact that all profiles start at the same bubble front. Furthermore they do not repeat themselves even at the slug unit distance away. Nevertheless some typical common behaviour of the temperature profile with time is observed. During the film passage the temperature always increases with time at a fixed location, while during the slug passage the temperature decreases part of the time and remains constant during the rest of it, in such a way that the temperature at the end of the period returns to its initial value.

Based on Fig. 2, it is evident that the time history of the liquid particles that start at the same relative location is not the same. As a result the temperature profile, $T(t)$, is not repeated at different X locations. Only in special cases where $X_p = t_{ms}V_s - t_{mf}V_f$ [the distance that a liquid particle passes between two similar locations relative to the Taylor bubble (Fig. 2)] is an integer function of the slug unit $L_B + L_s$ (or vice versa) the temperature variation will repeat itself at certain X locations. For example, in Fig. 5, X_p was chosen to be equal to $L_B + L_s$. In this case the temperature variation repeats itself every slug unit length.

Figure 6 shows the variation of the wall temperature with time for given locations along one slug unit for the case of constant heat flux. Step changes in the wall temperature are observed at the film–slug boundary as a result of the difference in the heat transfer coefficient.

Figure 7 shows the variation of the liquid temperature along the pipe at two different times which

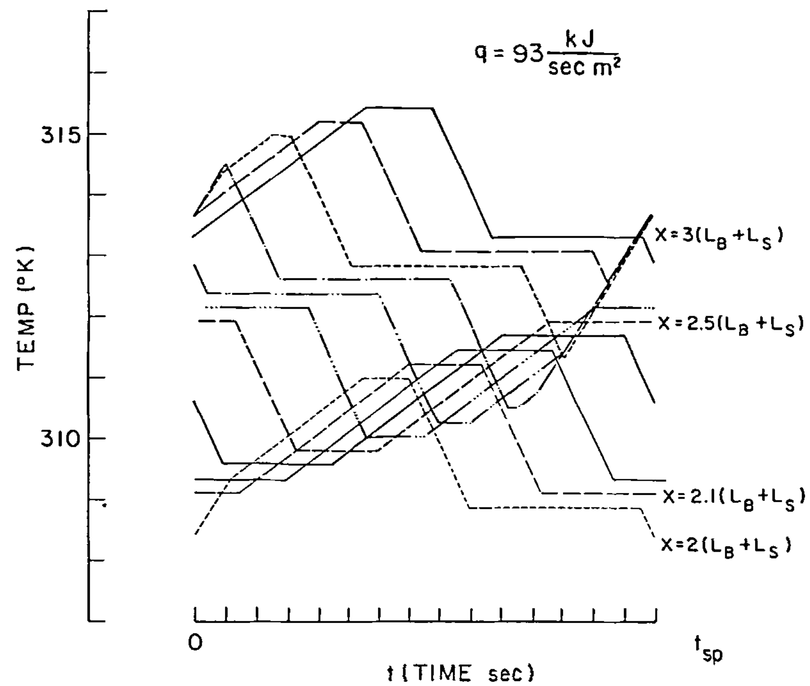
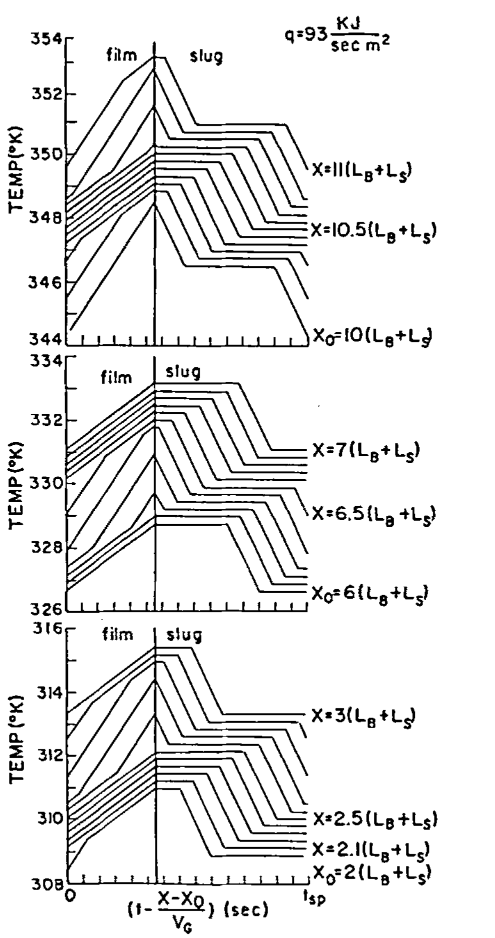


FIG. 3. Liquid temperature at various locations along a slug unit for the case of constant heat flux.



4. Liquid temperature vs $t-(X-X_0)/V_G$ for the case of constant heat flux.

differ by t_{sf}. The solid line represents a time for which a bubble front is located at $X = 0$, while the broken line represents a time for which a bubble tail is located at the entrance to the heated section. It is interesting to observe that although the average temperature increases linearly along the pipe (as in single phase flow) the instantaneous temperature may also decrease periodically. In the liquid slug the temperature increases with X while for the film the temperature decreases with X owing to the negative direction of the liquid film.

Figures 8 and 9 show the results obtained for the case of constant wall temperature. Figure 8 shows the variation of the liquid temperature with time at various positions along the heated section, while in Fig. 9 the variation of liquid temperature along the pipe is observed at two different times.

As expected for the case of constant wall temperature, the liquid temperature approaches the wall temperature for large X. Similar to the case of constant heat flux, the film temperature always increases with time (Fig. 8) while the slug temperature either decreases or remains constant with time. Likewise (Fig. 9) contrary to single phase flow where the temperature always increases along the pipe, here the monotonic general increases trend is composed of ups and downs 'zig-zag' sections.

The liquid temperature was calculated neglecting the heat transfer to the gas bubble. This assumption is quite reasonable owing to the very small heat transfer coefficient to the gas compared to the wall–liquid heat transfer coefficient. However, when the liquid temperature is known it is possible to calculate the gas temperature in a straightforward fashion. Following an

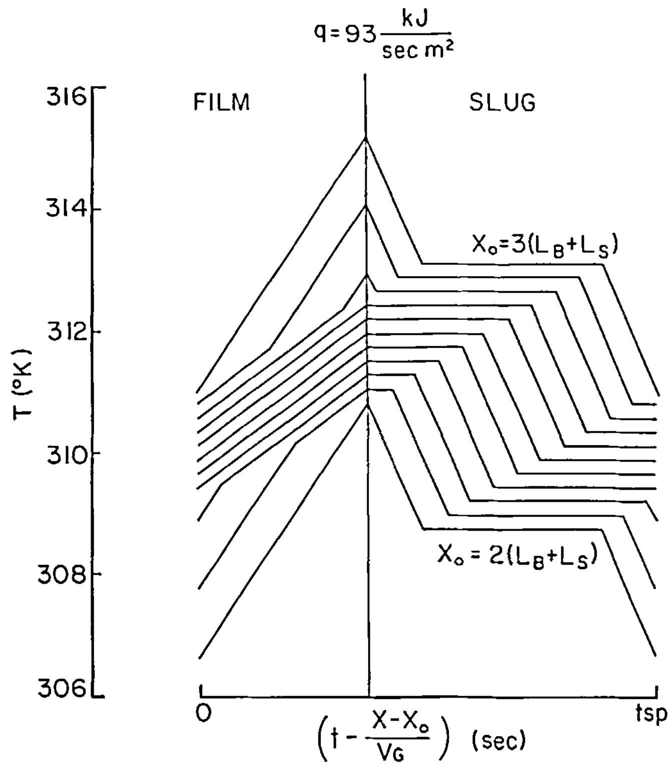


FIG. 5. Liquid temperature for the special case of $X_p = L_B + L_s$.

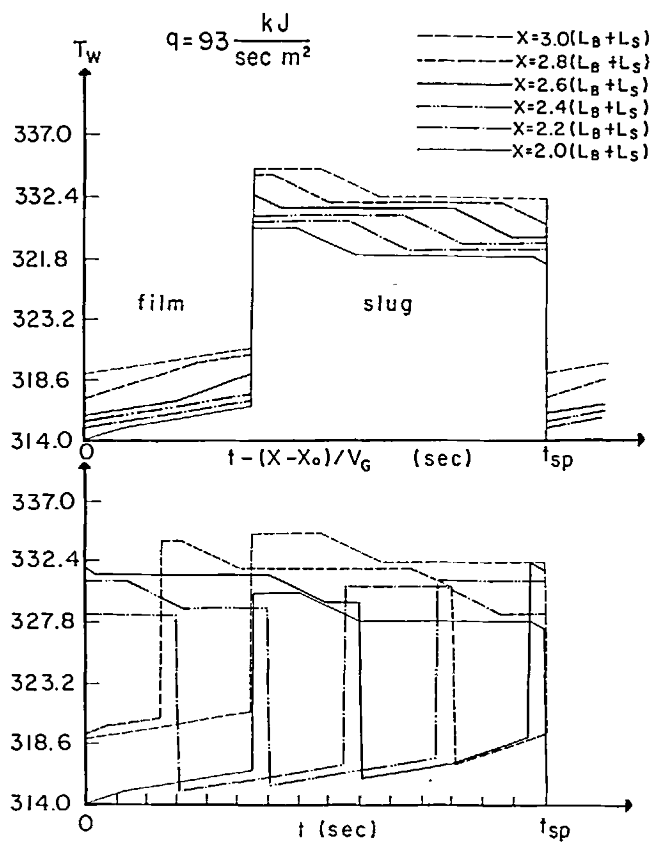


FIG. 6. Wall temperature at various locations along a slug unit for the case of constant heat flux.

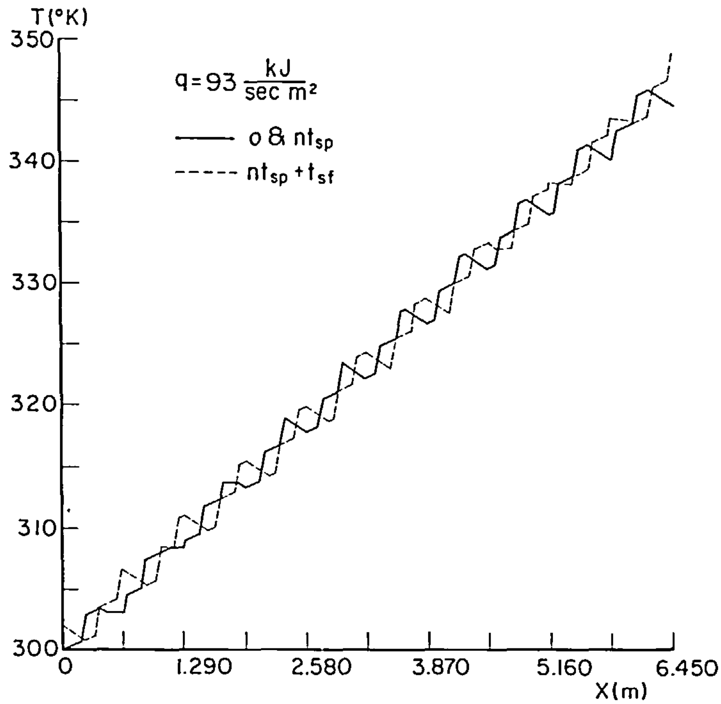


FIG. 7. Liquid temperature along the pipe. Constant heat flux.

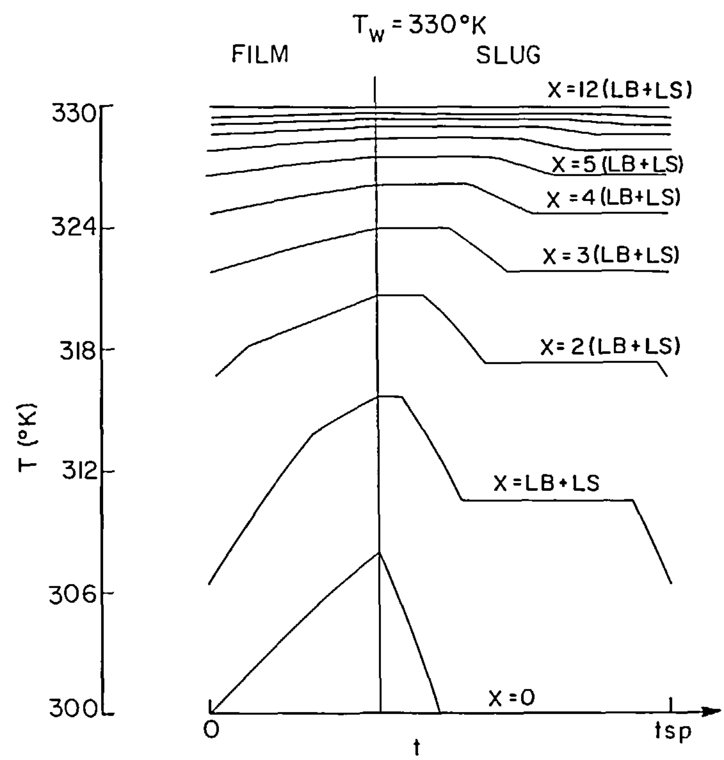


FIG. 8. Liquid temperature at various positions along the pipe for the case of constant wall temperature.

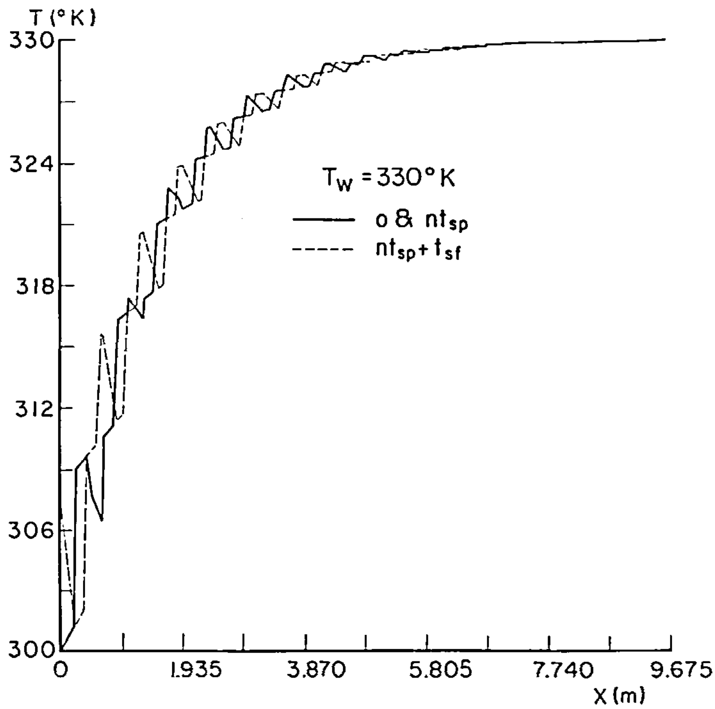


FIG. 9. Liquid temperature along the pipe. Constant wall temperature.

axial element of gas (Lagrangian coordinates) adjacent to the liquid film, its temperature rise with time is given by

$$\frac{dT_G}{dt} = \frac{4h_G}{\rho_G C_{pG} D_G}[T(X,t) - T_G] \tag{58}$$

where $X = V_G t + x$. x for $t = 0$ is the initial location of the gas element.

The solution of equation (58) for constant heat flux is given in Fig. 10 for two different positions in the Taylor bubble. In addition, the film temperature adjacent to the gas at the same positions is also given. As one would

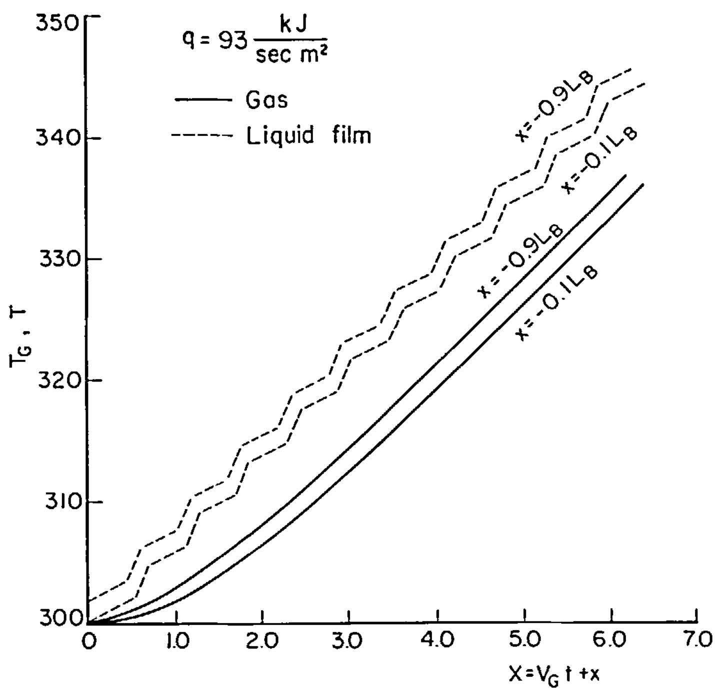


FIG. 10. Gas and liquid temperature along the pipe. Lagrangian coordinates.

expect the gas temperature increases while it rises in the pipe. The increase becomes linear at a distance of about 5 slug units from the entrance where average temperature difference between the gas bubble and its adjacent film is approximately constant.

Average heat transfer coefficients

The local average heat transfer coefficient over one slug period is defined by

$$\bar{h} = \frac{\bar{q}}{\bar{T}_w - \bar{T}} = \frac{\int_0^{t_{sp}} q \, dt/t_{sp}}{\int_0^{t_{sp}} T_w \, dt/t_{sp} - \int_0^{t_{sp}} T \, dt/t_{sp}}. \quad (59)$$

Equation (59) can be used directly to calculate numerically the average heat transfer coefficient using the results for the variation of the liquid and wall temperatures during one slug period at a preselected position.

For the case of constant heat flux, equation (59) takes the simple form of

$$\frac{1}{\bar{h}} = \left(\frac{1}{h_f} t_{sf} + \frac{1}{h_s} t_{ss}\right) \Big/ t_{sp}. \quad (60)$$

As seen, this average heat transfer coefficient is constant along the pipe and its inverse value is a time weighted average of the film and slug inverse heat transfer coefficients $(1/h)$.

For the case of constant wall temperature (59) takes the form

$$\bar{h} = \frac{T_w(h_f t_{sf} + h_s t_{ss}) - h_f t_{sf} \bar{T}_f - h_s t_{ss} \bar{T}_s}{t_{sp}(T_w - \bar{T})} \quad (61)$$

where

$$\bar{T}_f = \int_0^{t_{sf}} T \, dt/t_{sf},$$

$$\bar{T}_s = \int_{t_{sf}}^{t_{sp}} T \, dt/t_{ss}, \quad (62)$$

$$\bar{T} = \int_0^{t_{sp}} T \, dt/t_{sp}.$$

The numerical calculation of equation (62) shows that $\bar{T}_f \simeq \bar{T}_s \simeq \bar{T}$ relative to the temperature difference $T_w - \bar{T}$. In this case, equation (61) can be approximated by

$$\bar{h} = \frac{h_f t_{sf} + h_s t_{ss}}{t_{sp}}. \quad (63)$$

It is interesting to observe that the average heat transfer coefficient for constant heat flux is different than for the case of constant wall temperature. For the case of constant heat flux, the average heat transfer resistance $(1/h)$ is a linear weighted average of the film and slug heat transfer resistance while for the case constant wall temperature the heat transfer coefficient is a linear average of the film and slug heat transfer coefficients.

4. CONCLUSIONS

A mathematical model with an analytical solution based on the method of characteristics is derived for the unsteady heat transfer process in vertical gas-liquid slug flow.

The objective of this model is to gain physical insight related to the complex mechanism of heat transfer in two-phase slug flow. It provides a solution for the temperature of the liquid and the gas as a function of time and axial position and the wall temperature fluctuation. Solutions are presented for the case of constant heat flux and for the case of constant wall temperature. It is shown that the average heat transfer coefficient for the case of constant wall temperature is a linear weighted average of the film and slug heat transfer coefficients while for the case of constant heat flux the average heat transfer resistance $(1/h)$ is a linear average of the film and slug heat transfer resistances.

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TRANSFERT THERMIQUE DANS UN ECOULEMENT GAZ-LIQUIDE AVEC BOUCHONS, ASCENDANT, VERTICAL

Résumé—On présente une solution analytique pour le comportement variable du transfert thermique dans un écoulement gaz-liquide avec bouchons, ascendant, vertical. Les résultats donnent la prédiction de la variation de température avec le temps et la position, des fluctuations de la température de paroi, aussi bien que les coefficients moyens de transfert thermique.

WÄRMEÜBERGANG BEI VERTIKAL NACH OBEN GERICHTETER, ZWEIPHASIGER PFROPFENSTRÖMUNG

Zusammenfassung—Eine analytische Lösung für das zeitliche Verhalten des Wärmeübergangs bei vertikal nach oben gerichteteter, zweiphasiger Pfropfenströmung wird vorgestellt. Die Ergebnisse ermöglichen die Vorausberechnung der zeitlichen und örtlichen Temperaturänderungen, der Temperaturfluktuation an der Wand sowie des mittleren Wärmeübergangskoeffizienten.

ТЕПЛОПЕРЕНОС ПРИ ВЕРТИКАЛЬНОМ ВОСХОДЯЩЕМ СТЕРЖНЕВОМ ТЕЧЕНИИ ГАЗОЖИДКОСТНОГО ПОТОКА

Аннотация—Представлено аналитическое решение для описания неустановившегося теплопереноса при вертикальном восходящем стержневом течении газожидкостного потока. Результаты позволяют производить расчет изменений температуры в зависимости от координат, времени и флуктуации температуры стенки, а также расчет средних коэффициентов теплопереноса.